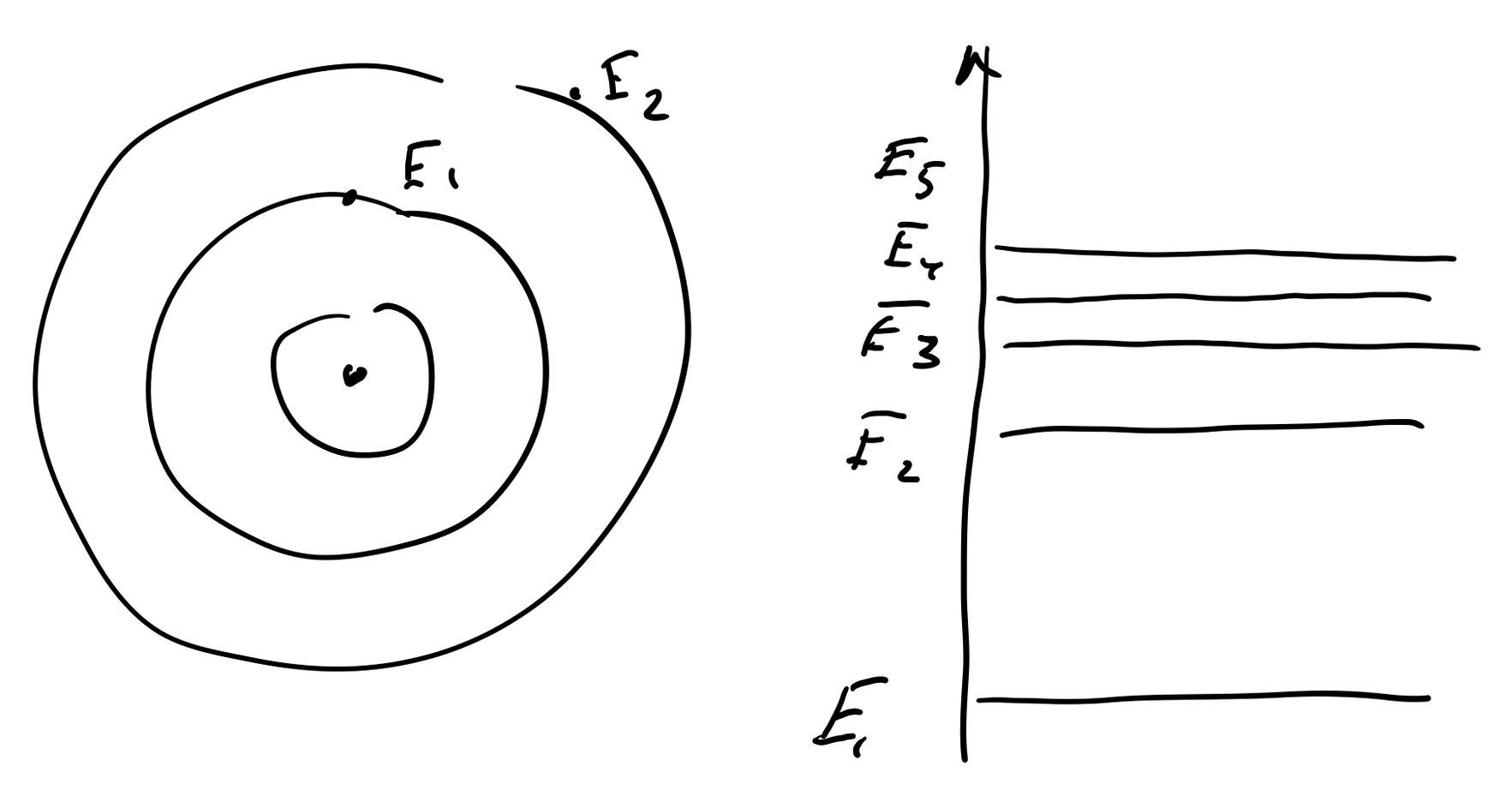
Let's derive Planck's formula based on Einstein approach.

① Bohr's postulates.

1) Electron in atom can be only in special states of special energy.



New model of atom – Bohr's model.

In stationary state electron energy does not change.

2) With atom transition from one state to another, the quantum of energy is absorbed or emitted. The energy of this quantum will be energy between states.

$$h\nu_{mn} = E_m - E_n$$

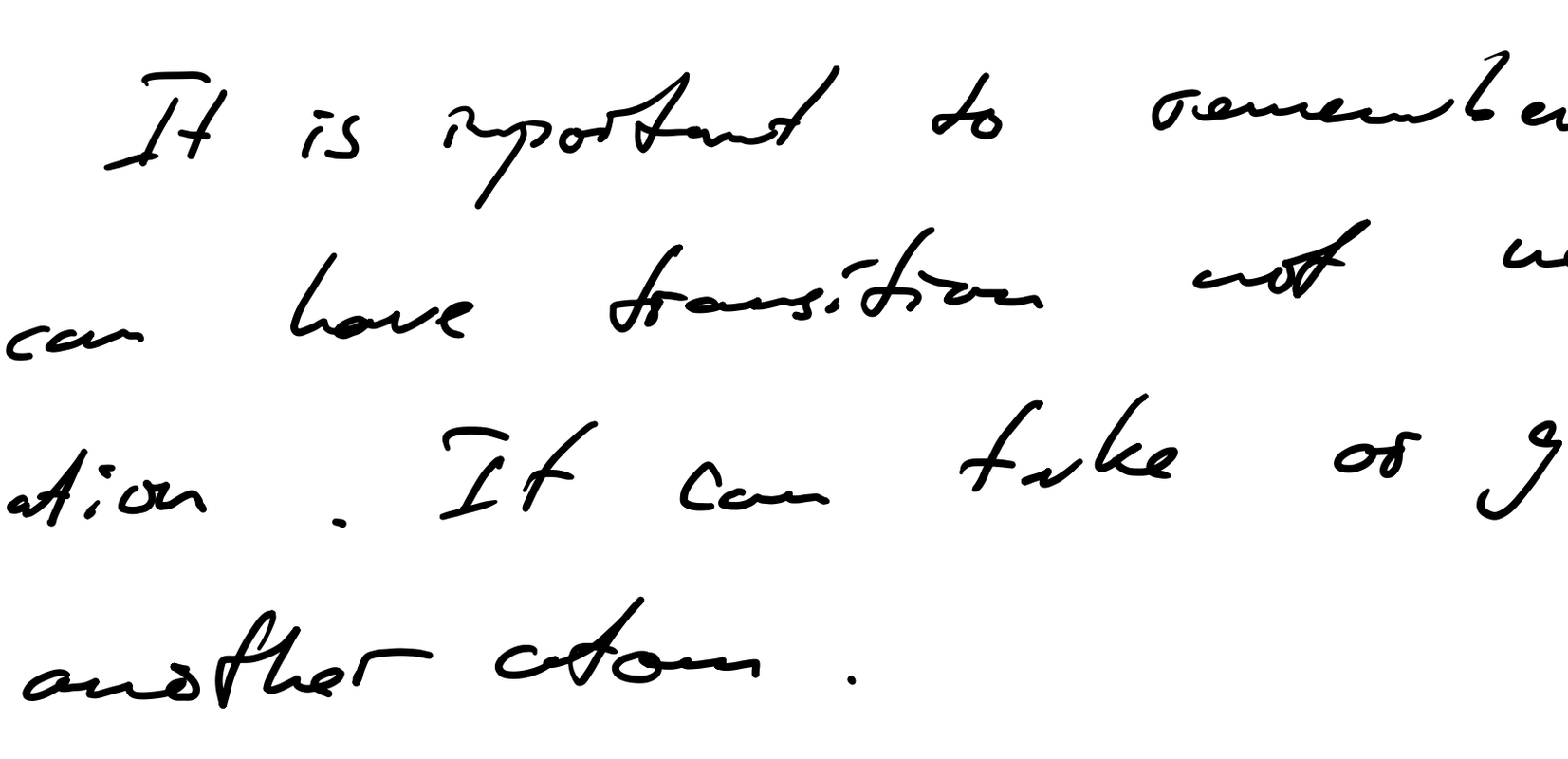
$h = 6.63 \cdot 10^{-34}$ J·s Planck's constant

② Optical transitions

Optical transitions are transitions of atoms from one level to another with absorption or emission of light.

It is important to remember, that atom can have transition not necessarily with radiation. It can take or give energy to another atom.

1) Spontaneous emission



$$dn_2 = -A_{21} n_2 dt$$

why?

n_2 is the concentration of atoms at 2.

⇓

$$n_2 = n_{20} e^{-A_{21} t} = n_{20} e^{-\frac{t}{\tau}} \qquad \tau = \frac{1}{A_{21}}$$

A_{21} is Einstein coefficient for spontaneous emission.

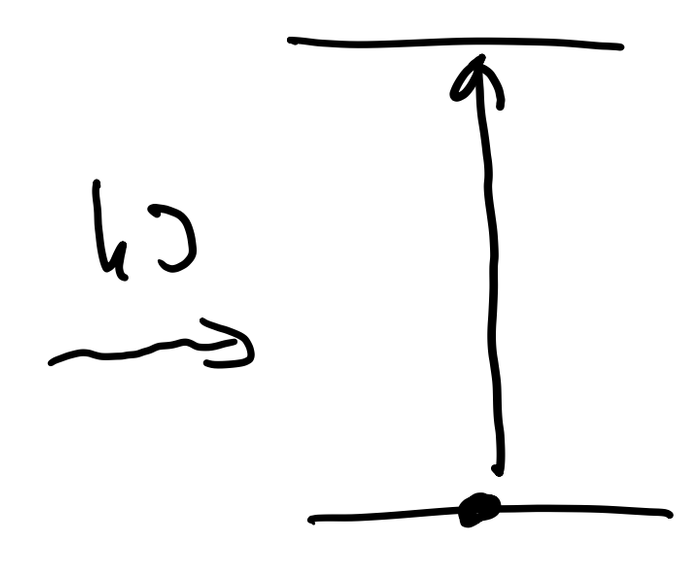
τ has clear physical meaning. It is time that takes to reduce concentration of particles at level 2 e times.

τ is called lifetime at the energy state.

Now we can explain hydrogen atom spectrum.

2) Absorption

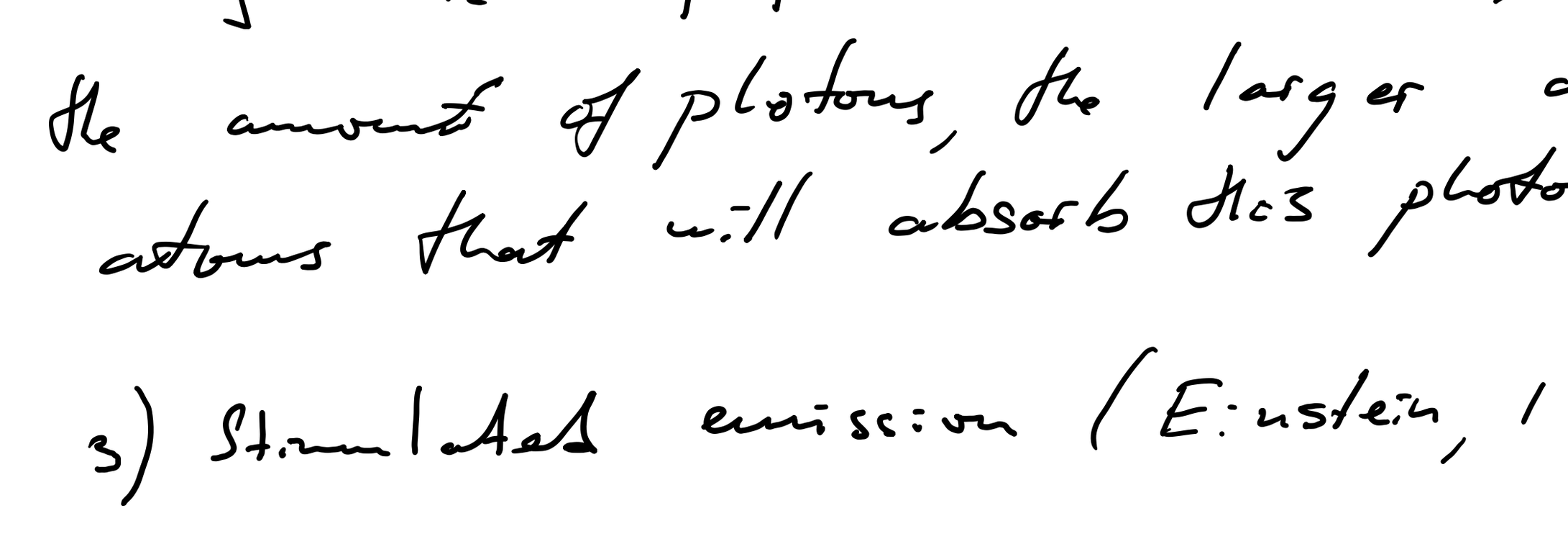
$$h\nu_{mn} = E_m - E_n$$



$$dn_{12} = B_{12} u(\nu, T) n_1 dt$$

Why dn_{12} is proportional to $u(\nu, T)$. The bigger the amount of photons, the larger amount of atoms that will absorb this photons.

3) Stimulated emission (Einstein, 1916)



In order to derive Planck's formula, Einstein guessed that there is one more type of emission, i.e. stimulated emission.

These photons have one frequency, one direction, one polarization.

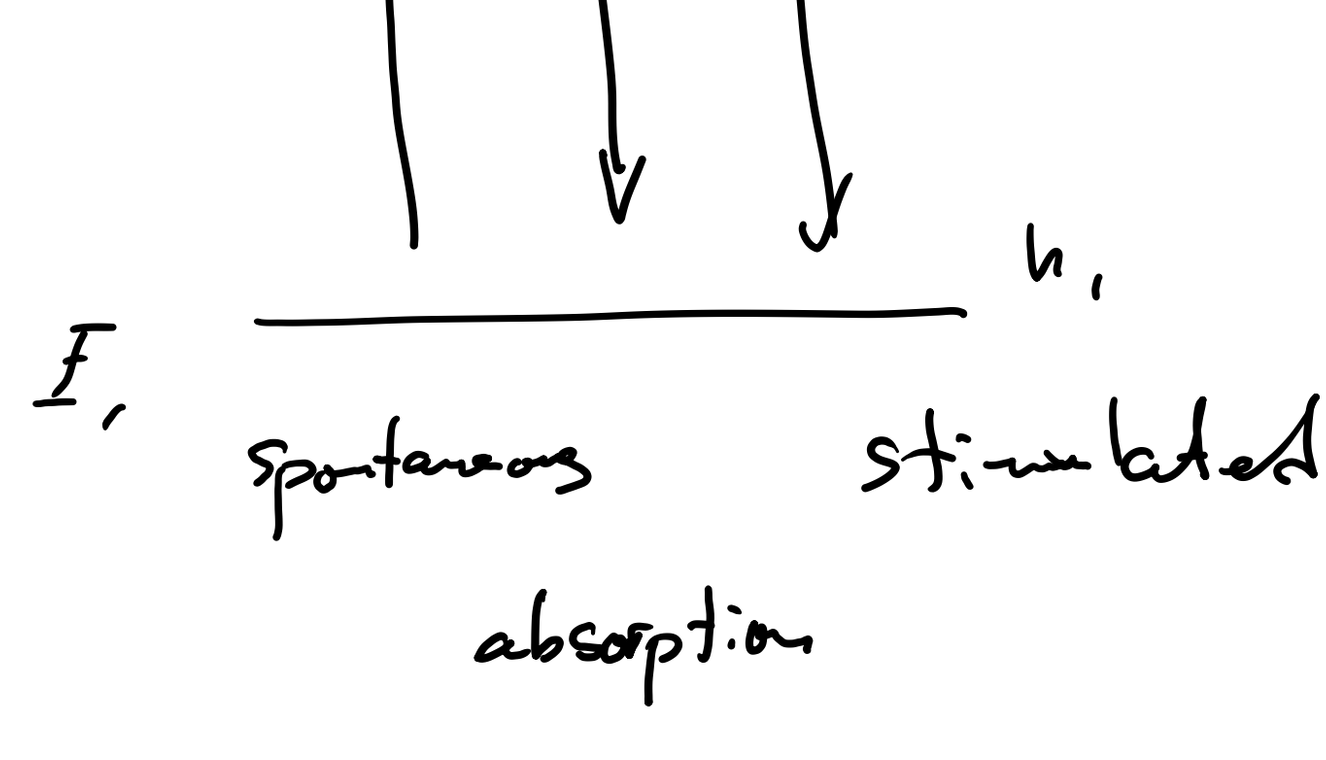
$$dn_{21} = B_{21} u(\nu, T) n_2 dt$$

A_{21}, B_{12}, B_{21} – Einstein coefficients for spontaneous emission, absorption and stimulated emission.

③ Planck's formula derivation

Under the thermodynamic equilibrium amount of transitions in 1 second from 1 to 2 is equal to 2 to 1.

$$h\nu = E_2 - E_1$$



$$n_{12} = n_{21}$$

$$B_{12} u(\nu, T) n_1 = A_{21} n_2 + B_{21} u(\nu, T) n_2$$

This is balance

$$n_1 = n_0 e^{-\frac{E_1}{kT}} \qquad n_2 = n_0 e^{-\frac{E_2}{kT}}$$

$$\frac{n_1}{n_2} = e^{\frac{E_2 - E_1}{kT}} = e^{\frac{h\nu}{kT}}$$

⇓

$$u(\nu, T) = \frac{A_{21}}{B_{12} e^{\frac{E_2 - E_1}{kT}} - B_{21}} = \frac{A_{21}}{B_{12} e^{\frac{h\nu}{kT}} - B_{21}}$$

From Stefan–Boltzmann law we know

$$T \to \infty \quad u(\nu, T) \to \infty \Rightarrow B_{12} = B_{21}$$

Hence,

$$u(\nu, T) = \frac{A_{21}}{B_{21}} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Now we need to determine A_{21} and B_{21}. We know, that for $h\nu \ll kT$ we have Rayleigh – Jeans formula.

$$e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT}$$

$$u(\nu, T) = \begin{cases} \frac{A_{21}}{B_{21}} \cdot \frac{kT}{h\nu} \\ \frac{8\pi\nu^2}{c^3} kT \end{cases} \Rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi\nu^3 h}{c^3}$$

(Rayleigh – Jeans)

⇓

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

For emission capacity

$$f(\nu, T) = \frac{c}{4} u(\nu, T) = \frac{2\pi\nu^2}{c^2} \cdot \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$